

MTH 435 Homework 1 - Answers - Solutions

1) (a) Equivalent (b) Equivalent (c) Not equivalent

2) (a) A tautology (b) Not a tautology (c) Not a tautology
(d) A tautology (e) A tautology

3) (a) False. If $x < 0$, $x \cdot y \geq 0$ requires $y \leq 0$. But then $x + y < 0$.

Negation: $\exists x \in \mathbb{R} \forall y \in \mathbb{R} (x \cdot y < 0 \text{ or } x + y \leq 0)$.

(b) True. For a given x take $y = x$. Then $x y z^2 = x^2 z^2 \geq 0$.

Negation: $\exists x \in \mathbb{R} \forall y \in \mathbb{R} \exists z \in \mathbb{R} x y z^2 < 0$.

(c) True. Take $y = 0$.

Negation: $\forall y \in \mathbb{R} \exists x \in \mathbb{R} x \cdot y \neq 0$

(d) True. Take $x = 1$.

Negation: $\forall x \in \mathbb{R} \exists y \in \mathbb{R} (y > 0 \text{ and } x y \leq 0)$

4) Denote open statements:

A: m is a positive integer, B: m^2 is divisible by 3

C: m is divisible by 3.

We are supposed to show $(A \wedge B) \Rightarrow C$. The "proof" begins with assuming C and ends with deducing C. It is proved that $C \Rightarrow C$ and not $(A \wedge B) \Rightarrow C$.

5) The proof is correct.

Denote statements:

A: π is irrational B: $\pi - x$ is irrational C: $\pi + x$ is irrational

We know A is true we want to show that $B \vee C$. The "proof" shows that $\neg(B \vee C) \Rightarrow \neg A$ by using the tautology $\neg(B \vee C) \Leftrightarrow \neg B \wedge \neg C$. Thus $\neg(B \vee C) \Rightarrow \neg A \wedge A$, Hence $B \vee C$.

Everything is correct.