

Exam II 243

1) $\int_0^1 \int_0^x \cos(x^2) dx dy =$ (6) (3)

$$= \int_0^1 \int_0^x 4 \cos(x^2) dy dx = \int_0^1 x \cos(x^2) dx = \frac{1}{2} \sin(x^2) \Big|_0^1 = \frac{1}{2} \sin(1) = 0.84/2 = 0.42$$

(4) (3)

2) $\int_1^2 \int_1^{2x} (2x^2 y) dy dx =$ (4)

$$\int_1^2 4x^2 (y^2 \Big|_{x}^{2x}) dx = \int_1^2 x^2 (4x^2 - x^2) dx =$$

$$= \int_1^2 3x^4 dx = \frac{3}{5} x^5 \Big|_1^2 = \frac{32 \cdot 3}{5} - \frac{3}{5} = \frac{93}{5} = 18.6$$

(6) (5)

3) $f_x = 3x^2 - 6x = 0 \quad f_y = 3y^2 - 3 = 0$

$$\begin{cases} y^2 - 1 = 0 \\ x^2 - 2x = 0 \end{cases} \rightarrow \begin{matrix} \pm 1 \\ 0, 2 \end{matrix}$$

Crit. pts. ~~(0,0), (0,1), (0,-1), (1,0), (1,1), (1,-1), (2,0), (2,1), (2,-1)~~
 $(0,1), (0,-1), (2,1), (2,-1)$

$f_{xx} = 6x - 6 \quad f_{yy} = 6y \quad f_{xy} = 0$

$D(x,y) = (6x-6)6y \quad D(0,1) < 0$ saddle $D(0,-1) > 0$, $f_{xx} < 0$
 \max

$(0,1)$, $(0,-1)$, $(2,1)$, $(2,-1)$
 saddle max min saddle

4) $f_x = 12x^3, \quad f_y = 6y^2, \quad (0,0), \quad f_{xx} = 36x^2, \quad f_{yy} = 12y, \quad D = 36 \cdot 12x^2y$
 $\underline{D(0,0)=0} \cdot \underline{5} \quad \text{if } D=0 \Rightarrow \text{corner} \quad \underline{-6}$

 ~~$\neq 0$~~ saddle

5) $f_x = 2xy, \quad f_y = x^2, \quad f_{xx} = 2y, \quad f_{yy} = 0 \quad f_{xy} = 2x \quad f(1,0) = 0$
 $(1,0): \quad 0 \quad 1 \quad 0 \quad 0$

$L(x,y) = f(1,0) + f_x(1,0)(x-1) + f_y(1,0)y = y \quad L(x,y) = y$

$Q(x,y) = L(x,y) + \frac{f_{xx}(1,0)}{2}(x-1)^2 + \frac{f_{yy}(1,0)}{2}y^2 + f_{xy}(1,0)(x-1)y = y + 2(x-1)y \quad \underline{Q(x,y) = y + 2(x-1)y}$

$L(0.9, 0.2) = 0.2 \quad Q(0.9, 0.2) = 0.16, \quad f(0.9, 0.2) = 0.162 \quad = \underline{\frac{2xy-y}{2}}$

$$6) z = (x+y)e^y, \quad x = g(t), \quad y = h(t)$$

$$\begin{aligned} \frac{\partial z}{\partial t} \Big|_{t=0} &= \frac{\partial z}{\partial x} \Big|_{(g(0), h(0))} \cdot \frac{dx}{dt} \Big|_{t=0} + \frac{\partial z}{\partial y} \Big|_{(g(0), h(0))} \cdot \frac{dy}{dt} \Big|_{t=0} = \\ &= \frac{\partial z}{\partial x} \Big|_{(2,0)} \cdot (-0.3) + \frac{\partial z}{\partial y} \Big|_{(2,0)} \cdot 2.5 = 1 \cdot (-0.3) + 3 \cdot (2.5) = \end{aligned}$$

$$\frac{\partial z}{\partial x} = e^y, \quad \frac{\partial z}{\partial y} = xe^y + e^y + ye^y = \underline{7.2}$$

$$(2,0) \rightsquigarrow \underline{1} \rightsquigarrow 2+1=\underline{3}$$

$$7) \quad \text{grad } G = y^2 \vec{i} + (-z + 2yx) \vec{j} + (2z - y) \vec{k}$$

$$(a) \quad \text{grad } G(1,2,3) = 4 \vec{i} + \vec{j} + 4 \vec{k} \quad \textcircled{4}$$

$$(b) \quad \| \text{grad } G(1,2,3) \| = \sqrt{16+1+1} = \sqrt{133} = 5.74 \text{ °F/m} \quad \textcircled{3}$$

$$(c) \quad (4 \vec{i} + \vec{j} + 4 \vec{k}) \cdot \left(\frac{3}{5} \vec{i} - \frac{4}{5} \vec{k} \right) = \frac{3}{5} - \frac{16}{5} = -\frac{13}{5} \text{ °F/m} = -2.6 \text{ °F/m} \quad \textcircled{3}$$

$$8) \quad F(x,y,z) = x^2 + y^2 + xyz, \quad \text{Grad } F = (2x-yz) \vec{i} + (2y-xz) \vec{j} - xy \vec{k}$$

$$\text{Grad } F(2,1,0) = 4 \vec{i} + 2 \vec{j} - 2 \vec{k} \quad \text{- normal (a)} \quad \textcircled{6}$$

Noeg: $\textcircled{3}$

$$(b) \quad 4(x-2) + 2(y-1) - 2z = 0 \quad 2z = 4x + 2y - 10, \quad z = \underline{2x+y-5} \quad \textcircled{4}$$

$$9) \quad \text{grad } g = (2x + y \cos(xy)) \vec{i} + x \cos(xy) \vec{j}$$

$$\text{grad } g(2,0) = 4 \vec{i} + 2 \vec{j} \quad (a) \quad g \vec{j}(2,0) = \frac{4}{\sqrt{10}} - \frac{6}{\sqrt{10}} = -\frac{2}{\sqrt{10}} = -0.63$$

$$(b) \quad (2,0) \vec{(2,3)} = 3 \vec{j} \quad \vec{u} = \vec{j} \quad g \vec{j}(2,0) = \underline{2} \quad \textcircled{4}$$

$$10) \quad T(2,2,2,7) = 75 + T_x(2,3) \cdot 0.2 + T_y(2,3) (-0.3) =$$

$$= 75 + (-18.2) \cdot 0.2 + 14.5 (-0.3) = \underline{67.01 \text{ °C}}$$

Extraw: (a) $T := (x, y) \rightarrow 2/(x^2 + y^2 + 1);$

(b) $\text{grad } T := \text{grad } (T(x,y), [x,y]);$

(c) $\text{fieldplot } (\text{grad } T, x=-5..5, y=-5..5);$