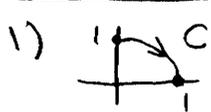


# Exam III - Solutions



$$\vec{F}(x,y) = (x+y)\vec{i} + y\vec{j}$$

The standard parametrization of the unit circle is ccw. Hence:

$$-C: \vec{r}(t) = \cos(t)\vec{i} + \sin(t)\vec{j}, \quad t \in [0, \frac{\pi}{2}]$$

Hence:

$$\begin{aligned} \int_C \vec{F} d\vec{r} &= - \int_{-C} \vec{F} d\vec{r} = - \int_0^{\frac{\pi}{2}} ((\cos(t)+\sin(t))\vec{i} + \sin(t)\vec{j}) \cdot (-\sin(t)\vec{i} + \cos(t)\vec{j}) dt = \\ &= - \int_0^{\frac{\pi}{2}} \sin^2 t dt = \underline{\underline{\frac{\pi}{4} \approx 0.785}} \end{aligned}$$

2)  $\vec{F} = xy\vec{i} + (x-y)\vec{j}$ ,  $C: (0,1) \rightarrow (1,0)$

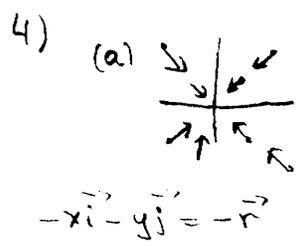
$C: x = t, y = 1-t, t \in [0,1]$

$$\int_C \vec{F} d\vec{r} = \int_0^1 ((t(1-t))\vec{i} + (2t-1)\vec{j}) \cdot (\vec{i} - \vec{j}) dt = \int_0^1 (-t^2 - t + 1) dt = \underline{\underline{\frac{1}{6} = 0.1666}}$$

3) Let  $C$  be the ellipse  $4x^2 + y^2 = 4$  or  $x^2 + \frac{y^2}{4} = 1$  oriented ccw. Then

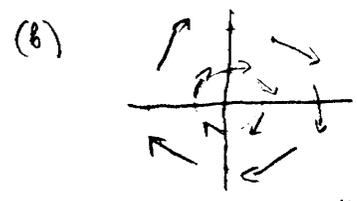
$C: x = \cos t, y = 2 \sin t, t \in [0, 2\pi]$

$$\begin{aligned} \oint_C \vec{F} d\vec{r} &= \int_0^{2\pi} (e^{\cos t}\vec{i} + e^{2\sin t}\vec{j}) \cdot (-\sin t\vec{i} + 2\cos t\vec{j}) dt = \\ &= \int_0^{2\pi} (-\sin t e^{\cos t} + 2\cos t e^{2\sin t}) dt = (e^{\cos t} + e^{2\sin t}) \Big|_0^{2\pi} = \underline{\underline{0}} \end{aligned}$$

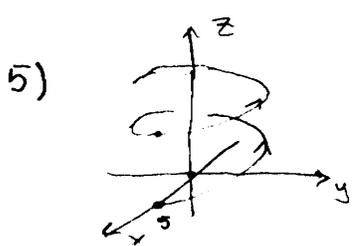


The vectors point toward the origin; their magnitude is constant on each circle centered at the origin and increases as we move away from the origin.

$$-x\vec{i} - y\vec{j} = -\vec{r}$$



Vectors are tangent to circles centered at the origin pointing ccw. Indeed,  $y\vec{i} - x\vec{j} \perp x\vec{i} + y\vec{j}$ . Magnitude increases as we move away from the origin.



The body is traveling along a helix as pictured.

$$\vec{v}(t) = -5\sin(t)\vec{i} + 5\cos(t)\vec{j} + 5\vec{k}$$

$$\vec{a}(t) = -5\cos(t)\vec{i} - 5\sin(t)\vec{j}$$

The body is 10 m over ground at  $t=2$ .

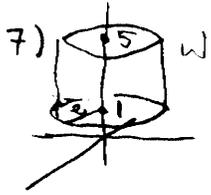
$$\vec{v}(2) = -5\sin(2)\vec{i} + 5\cos(2)\vec{j} + 5\vec{k} = \underline{\underline{-4.55\vec{i} - 2.08\vec{j} + 5\vec{k}}}$$

$$\vec{a}(2) = -5\cos(2)\vec{i} - 5\sin(2)\vec{j} = \underline{\underline{2.08\vec{i} - 4.55\vec{j}}}$$

$$\text{Speed}(2) = \|\vec{v}(2)\| \hat{=} \underline{\underline{7.073 \frac{m}{min}}}$$

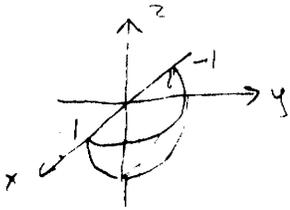
6) (a)  $x = 2 + 3t$ ,  $y = 3 - t$ ,  $z = -1 + t$ ,  $t \in (-\infty, \infty)$

(b) The lines are parallel as  $\vec{v} = 3\vec{i} - \vec{j} + \vec{k}$  and  $\vec{w} = -6\vec{i} + 2\vec{j} - 2\vec{k}$  are parallel.



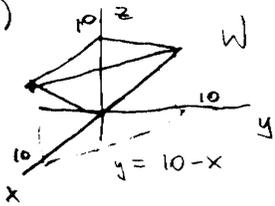
$$\int_0^{2\pi} \int_0^2 \int_1^5 r \cdot r \cdot dz \cdot dr \cdot d\theta = \int_0^{2\pi} \int_0^2 4r^2 \cdot dr \cdot d\theta = \int_0^{2\pi} \left( \frac{4}{3} r^3 \Big|_0^2 \right) d\theta = \frac{64\pi}{3} = 67.02$$

8)



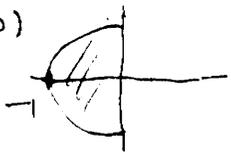
The region is the quarter of the unit sphere below the xy-plane to the right of the xz plane.

9)



$$\text{vol}(W) = \int_W 1 \, dV = \int_0^{10} \int_0^{10-x} \int_0^{10-x-y} dz \, dy \, dx = \frac{500}{3} \approx 166.66$$

10)



$$\int_{-1}^0 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x \cdot y \, dx = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^1 r \cos \theta \cdot r \, dr \, d\theta = \underline{\underline{-\frac{2}{3} = -0.66...}}$$