Section 3.9: Linear Approximation and the Derivative

The Tangent Line Approximation

\[ f(x) \approx f(a) + f'(x)(x - a) \]

\[ E(x) = f(x) - f(a) - f'(x)(x - a) \]

EX: What is the tangent line approximation for \( f(x) = \sin x \) near \( x = 0 \)?
EX: What is the local linearization of $e^{kx}$ near $x = 0$?

**Problem 1** Find the local linearization of $f(x) = x^2$ near $x = 1$.

**Problem 2** Find the tangent line approximation for $\sqrt{1 + x}$ near $x = 0$.

**Problem 4** Find the tangent line approximation for $\frac{1}{x}$ near $x = 1$. 
Problem 6 Show that $e^{-x} \approx 1 - x$ near $x = 0$.

Problem 10 For $x$ near 0, local linearization gives $e^{-x} \approx 1 - x$. Using a graph, decide if the approximation is an overestimate or underestimate, and estimate to one decimal place the magnitude of the error for $-1 \leq x \leq 1$. 
Problem 12

(a) Graph \( f(x) = x^3 - 3x^2 + 3x + 1 \).

(b) Find and add to your sketch the local linearization to \( f(x) \) at \( x = 2 \).

(c) Mark on your sketch the true value of \( f(1.5) \), the tangent line approximation to \( f(1.5) \) and the error in the approximation.
Problem 14

(a) Show that $1 + kx$ is the local linearization of $(1 + x)^k$ near $x = 0$.

(b) Someone claims that the square root of 1.1 is about 1.05. Without using a calculator, do you think that this estimate is about right?

(c) Is the actual number above or below 1.05?

Problem 16 The equation $e^x + x = 2$ has a solution near $x = 0$. By replacing the left side of the equation by its linearization, find an approximate value for the solution.
Problem 34

(a) Show that $1 - x$ is the local linearization of $\frac{1}{1 + x}$ near $x = 0$.

(b) From your answer to part (a), show that near $x = 0$, $\frac{1}{1 + x^2} \approx 1 - x^2$

(c) Without differentiating, what do you think the derivative of $\frac{1}{1 + x^2}$ is at $x = 0$?