Practice Problems for Test 3 - MTH 141 Fall 2003

Sections 4.1, 4.2, 4.3, 4.5, 4.6, 4.7, 5.1, 5.2, 5.3, 5.4, 6.1

NOTE: This is a selection of problems for you to test your skills. The idea is that you get a taste of the *kind* of problems that may appear on tests. Of course, some problems in the actual test may be completely different to the problems found here. Further, the questions in this document may not represent all the material discussed in class. To be better prepared for the test, make sure you review assignments, quizzes, class notes, and read the book.

For each function in items (1) through (4), and without referring to a plot, (a) find all the critical points of the function, (b) classify the critical points with the first derivative or second derivative test, and (c) find all points x that satisfy $f''(x) = 0$, and determine if they are inflection points with a test.

1. $f(x) = 2x^3 + 3x^2 - 72x$
2. $g(x) = 3x^4 + 8x^3 - 6x^2 - 24x + 2$
3. $h(t) = -te^{-t^2}$
4. $R(t) = -1 + 2\ln(t^2 + 2t + 2)$

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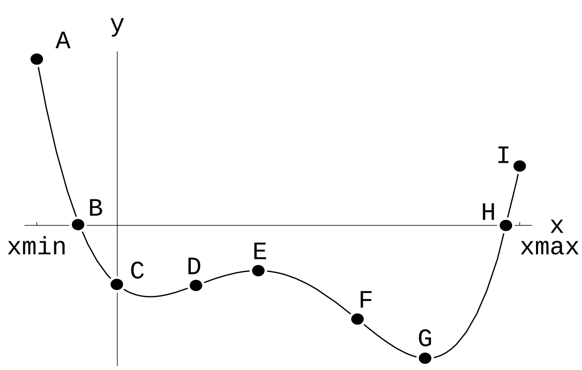
5. The plot of a function defined on the interval $[x_{min}, x_{max}]$ is shown. Fill in the table.



Kind of point	Point or Points
Critical point	
Inflection point	
Local min	
Local max	
Global min	
Global max	

6. Find the constants a and b in the function $f(x) = axe^{bx}$ such that $f(\frac{1}{3}) = 1$, and the function $f(x)$ has a local maximum at $x = \frac{1}{3}$.

7. Let $f(x) = x^2 + 2ax$. Find all x-intercepts, y-intercepts, critical points. Classify critical points with a test. Sketch the graph of $f(x)$, and label all important points (in terms of "a" if necessary). What effect does increasing the value of "a" has on the graph of the function?

8. Let $f(x) = 2x^3 + 3x^2 - 72x$. Find the x-values where $f(x)$ has a global maximum and where it has a global minimum on each of the intervals given below. Justify your answer without referring to a plot.

   (a) $0 \leq x \leq 4$ (b) $-5 \leq x \leq 4$ (c) $-2 \leq x \leq 2$ (d) $4 \leq x \leq 10$

9. Let $h(t) = -te^{-t^2}$. Find the t-values where $f(t)$ has a global maximum and where it has a global minimum on each of the intervals given below. Justify your answer without referring to a plot.

   (a) $-2 \le t \le 0$ (b) $-2 \le t \le 2$ (c) $-0.5 \le t \le 0.5$ (d) $-3 \le t \le -1$

10. Find the best possible bounds for the function $f(x) = xe^{-x}$ on the interval $0.5 \le x < \infty$.

11. A 2880 square yard rectangular region will be surrounded with fencing that costs \$ 5 per yard on three sides, and a fancier fencing that costs \$ 7.5 per yard will be used on the fourth side. Find the dimensions of the rectangle of minimal cost, and also find the minimal cost.

12. Of all the rectangles with given area A, which has the shortest diagonals?

13. Use the definition of $\cosh t$ and $\sinh t$ in terms of exponential functions to prove that $(\cosh t)' = \sinh t$.

14. Compute the derivative of $\tanh x$.

15. In section 4.7 we studied the Extreme Value Theorem, the Mean Value Theorem, the Increasing Function Theorem, and the Constant Function Theorem. Say which theorem justifies each of the statements below:

   A The function $f(x)$ has zero derivative on the interval $-1 < x < 1$, hence $f(x) = f(0)$ for all x in the interval $-1 < x < 1$.

   B The object moving along a line has positive velocity, hence its position $s(t)$ increases with time t.

   C The object moving along a line was initially at $s = 0$ feet, and 10 seconds later it was at $s = 20$ feet. Therefore, there is at least one time t when the instantaneous velocity was 2 feet per second.

   D A motorcycle moving for $0 \le t \le 1$, t in hours, along a straight road has continuous instantaneous velocity. Therefore it reaches its maximum velocity at some particular time during the one hour time interval.

16. Velocity data for an airplane that moves in a straight line is shown in the table below. Suppose the velocity is increasing all the time.

t (hours)	0.0	0.2	0.4	0.6	0.8	1.0
v (mi/hour)	150	170	200	240	290	350

   a) Find upper and lower estimates for the total distance traveled by the airplane between $t = 0.2$ and $t = 0.8$.

   b) How often do we need to measure speed in order for the gap between upper and lower estimates of distance traveled between $t = 0.2$ and $t = 0.8$ is less than 0.1 miles ? (That is, what should Δt be?)

   c) How many subdivisions of the interval $0.2 \le t \le 0.8$ do we need (that is, what is n) to get the Δt found in part (b)?

17. Polluted water enters a lake through a river. It is known that the amount of pollutants $Q(t)$ tons at time t months on the lake has been increasing. The following table shows measurements of the rate at which pollutants enter the lake, where t is the number of months after March 15, 2001.

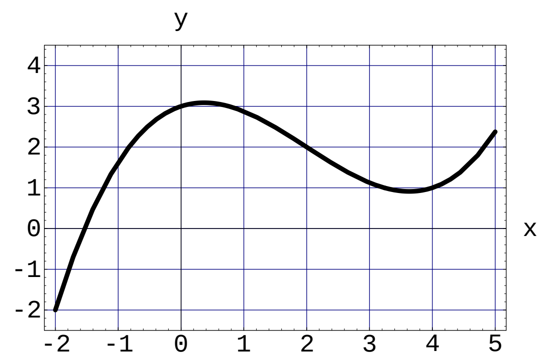
t (months)	0	3	6	9	12	15
$\frac{dQ}{dt}$ (tons/month)	0.25	0.19	0.10	0.06	0.02	0.01

a) Obtain an underestimate and an overestimate of the total amount of pollutants that entered the lake during the time period $3 \leq t \leq 15$.

b) How often would measurements have to be made to find overestimates and underestimates which differ by less than 0.0001 tons from the exact quantity of pollutants that entered the lake during $3 \leq t \leq 15$? What is the total number of measurements made in this case?

18.

For the function with plot shown, and for $-2 \leq t \leq 4$, find as best you can



a) Left-hand sum with $\Delta t = 2$
b) Right-hand sum with $\Delta t = 2$
c) Left-hand sum with $\Delta t = 3$
d) Right-hand sum with $\Delta t = 3$
e) Left-hand sum with $n = 3$.
f) Left-hand sum with $n = 2$.

19. a) Use the table to get a reasonable approximation to of $\int_0^{10} f(t)dt$.
b) What is the average value of $f(t)$ on $0 \leq t \leq 10$?

t	0	2	4	6	8	10
$f(t)$	-3.5	-1.7	-0.5	0.8	2.1	1.8

20. a) Compute the Left sum and right sum approximations to $\int_0^{10} -x^2 + 10x dx$ with $n = 6$. Verify that answers are equal.

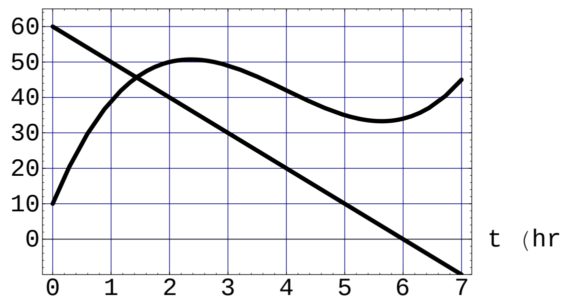
b) Explain this apparent coincidence. Can you conclude that in our example $\int_0^{10} -x^2 \ + \ 10x \ dx = \text{LEFT SUM} = \text{RIGHT SUM}$?

21. Use $n = 3$ to find the left-hand sum and right-hand sum approximations to $\int_0^5 \sin(x^2)dx$. Show the sums with all their terms first, and then calculate their numerical value.

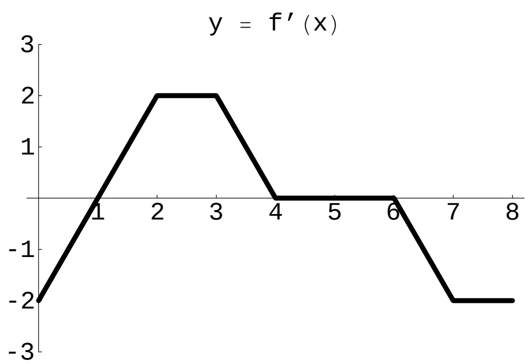
22. Use $\Delta t = 0.2$ to find the left-hand sum and right-hand sum approximations to $\int_{-1}^0 e^{-t^2} dt$. Show the sums with all their terms first, and then calculate their numerical value.

23.

A car and a truck start together at time $t = 0$ hr as they move along a straight road. The plots of their velocities (in mi/hr) are shown in the figure, where the car's velocity is the one that decreases linearly.
a) When are the vehicles separated the most?
b) Do both vehicles ever meet again during the first 7 hours? If so, when, and if it doesn't, why not.
c) Describe in words what is happening at times $t = 6$ and $t = 7$ hours.

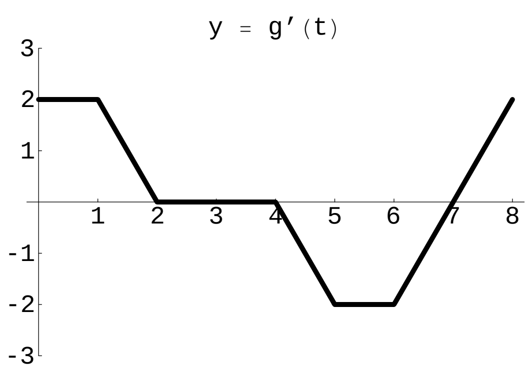


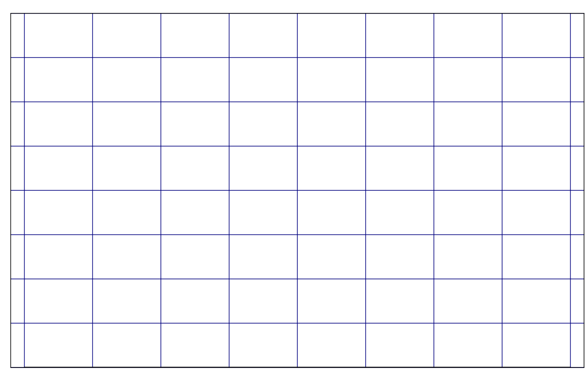
24. The following graph shows the derivative $f'(x)$ of a function. Use the graph to fill in the table values of $f(x)$, given that $f(0) = 1$.



x	0	1	2	3	4	5	6	7	8
$f(x)$									

25. The graph on the left shows the derivative $g'(t)$ of a function. Plot $g(t)$, given that $g(0) = -1$. (Make sure you label tickmarks in the horizontal and vertical directions.)





26. Suppose that $f(t)$ and $g(t)$ are functions for which $\int_0^5 f(t)dt = 23$, $\int_0^5 g(t)dt = 15$, $\int_3^5 f(t)dt = 13$, $\int_3^5 g(t)dt = 22$.

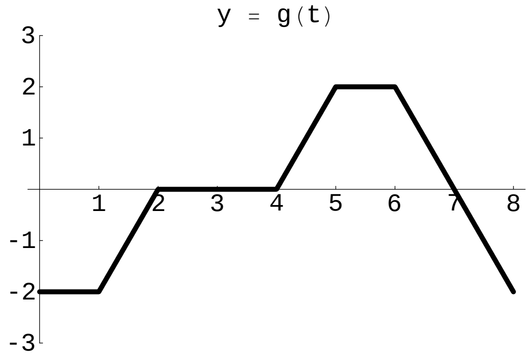
a) Find $\int_0^5 3f(t) + 2g(t)\, dt$, or explain why it cannot be found with the given information.

b) Find $\int_0^2 f(t)\, dt$ or explain why it cannot be found with the given information.

c) Find $\int_0^3 g(t)\, dt$ or explain why it cannot be found with the given information.

d) Find $\int_5^0 g(t)\, dt$ or explain why it cannot be found with the given information.

27. The following figure shows a function the graph of $g(t)$. Use the graph to fill in the table values of the antiderivative of $G(t)$ for which $G(0) = 1$.



t	0	1	2	3	4	5	6	7	8
$G(t)$									

28. The following figure shows the graph of a function $f(t)$. Plot the antiderivative $F(t)$ of $f(t)$, given that $F(0) = 0$. (Make sure you label tickmarks in the horizontal and vertical directions.) Also, give the coordinates of critical points and of inflection points.