Practice Problems for Test 2 - MTH 141 Fall 2003
Sections 3.1 — 3.10

NOTE: This is a selection of problems for you to test your skills. The idea is that you get a taste of the *kind* of problems that may appear on tests. Of course, some problems in the actual test may be completely different to the problems found here. Further, the questions in this document may not represent all the material discussed in class. To be better prepared for the test, make sure you review assignments, quizzes, class notes, read the book, etc.

PART 1: CALCULATOR NOT ALLOWED WHILE THIS PAPER IS IN YOUR POSESSION. You may work on both parts of the test, but in order to use a calculator, you have to turn in this part of the test.

Compute the derivative of the given functions. Below A, B, R, and S are constants.

$$a(x) = -3x^2 + \frac{1}{x^5} - \frac{2}{3}\sqrt{x} \qquad\qquad b(t) = -7e^t + 2^t - \left(\frac{1}{2}\right)^t$$

$$c(x) = 5\ln x - 3\cos(x) + 2\tan(x) \qquad\qquad d(t) = \frac{3t^3 - 5t^2}{1 - 4t^3}$$

$$e(x) = \frac{7}{\cos(x)} - x\sin(x) \qquad\qquad f(t) = \arctan(5t - 2)$$

$$g(x) = \sin^2(x) - \sin(2x) \qquad\qquad h(u) = \ln\left(2^u + 3\right)$$

$$i(x) = (\ln(x) + 2)^{0.7} \qquad\qquad j(v) = \sqrt[3]{ve^{2v+1}}$$

$$k(\theta) = \sqrt{3 - \sqrt{5 - \theta}} \qquad\qquad l(t) = \frac{3 + t}{\sqrt{t + 2}} + \sqrt{\frac{3 + t}{t + 2}}$$

$$m(x) = \sqrt{\tan(3x)} \qquad\qquad n(t) = e^{Kt}(\cos(Rt) + \sin(St))$$

$$o(\phi) = \frac{1 - a\phi}{1 + a\phi} \qquad\qquad p(\psi) = \sqrt{A^2 - \sin^2(\psi)}$$

$$q(t) = (At^2 + B)e^{Ct} \qquad\qquad r(x) = \sqrt{\pi} + e^2$$

$$s(x) = e^{\sin x}\cos x \qquad\qquad t(\theta) = \frac{\sin(A\theta)}{1 - \sin(A\theta)}$$

$$u(x) = x\arcsin(\frac{x}{A}) \qquad\qquad v(x) = \frac{1}{3x} - x2^{-x}$$

$$w(x) = 3^{x\ln x} + (x\ln x)^3 \qquad\qquad x(\theta) = (\ln\theta)^3 - \ln(\theta^3)$$

$$y(\mu) = -2A\mu\sqrt[3]{1 - A\mu^2} \qquad\qquad z(\eta) = A\eta^2 e^{B\eta^2}$$

Suppose that f and g are differentiable with the values given in the table:

t	$f(t)$	$g(t)$	$f'(t)$	$g'(t)$
5	-2	4	6	11
-2	6	17	13	3
4	23	21	-1	7

For the table given above, and for each of the following functions $h(t)$, find $h'(5)$:

(1) $h(t) = g(f(t))$ (2) $h(t) = f(g(t)$ (3) $h(t) = g(g(t))$

(4) $h(t) = f(t)g(t)$ (5) $h(t) = \frac{f(t)}{g(t)}$ (6) $h(t) = \ln f(t)$

In problems (7.)–(10.), use implicit differentiation to compute $\frac{dy}{dx}$.

(7.) $y^2 - x^2y^2 = 6$ (8.) $xe^y - y = 0$

(9.) $\sin(ax) + \cos(ay) = 1$ (10.) $\frac{x}{y} - \frac{1}{x} = 5y + 1$

(11.) Consider the curve with equation $y^2 + 2 = xy$.
 (a.) Verify that the point $P = (3, 1)$ is on the curve.
 (b.) Obtain the equation of the tangent line to the curve at $P = (3, 1)$.
 (c.) Find all points (x, y) on the curve where the tangent line is horizontal.
 (d.) Find all points (x, y) on the curve where the tangent line is vertical.

(12.) Parametrize each one of the following curves:
 (a.) A circle, traversed once counterclockwise, centered at $(-2, 5)$ and with radius 3.
 (b.) A line segment from $(-8, 3)$ to $(5, 2)$.
 (c.) The section of the graph of $y = x^3 + x$ between $(-2, -10)$ and $(1, 2)$.
 (d.) A line through $(-2, 0)$ and $(0, -4)$.

(13.) A particle moves on the XY plane according to the equations
 $x = (t - 1)^3$, $y = -2(t - 1)^2$, for $t \geq 0$.
 (a.) Find the speed at time t, and at time $t = 2$.
 (b.) Is the particle moving "up" or "down" when $t = 2$? Why?
 (c.) What is the slope of the path described by the particle when $t = 2$?
 (d.) Does the particle ever come to a stop? When?

(14.) Given $y = f(x)$ with $f(1) = 4$ and $f'(1) = 3$,
  (a.) find $g'(1)$ if $g(x) = \sqrt{f(x)}$ (b.) find $h'(1)$ if $h(x) = f(\sqrt{x})$

In problems 15–18, compute the tangent line approximation (local linearization) of the given function about the given point.

(15.) $f(t) = e^{1-2t}$, about 0.

(16.) $g(t) = \arctan(t^2)$, about 0.

(17.) $h(x) = \sin(x)\cos(x)$, about $\frac{\pi}{4}$.

(18.) $m(x) = 5x + 4$, about 1.

(19.) If the linearization is used to approximate $f(0.02)$ in problem (15.), find the error.

(20.) If the linearization is used to approximate $h(0.79)$ in problem (17.), find the error.

Compute the following limits. Use L'Hopital's rule if appropriate, and in this case label the limit as being of type "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$".

(21.) $\lim\limits_{x\to 0} \dfrac{e^{4x} - 1}{\sin(3x)}$

(22.) $\lim\limits_{x\to 0} \dfrac{(\ln x)^2}{x^3 - 2x^2 + x}$

(23.) $\lim\limits_{x\to\infty} \dfrac{x^2}{e^{-x^2} + 3x^2}$

(24.) $\lim\limits_{x\to 0} \dfrac{x}{e^{-x^2}}$

(25.) $\lim\limits_{x\to 0} \dfrac{\cos(x)}{x^2}$

(26.) $\lim\limits_{t\to\pi} \dfrac{1 + \cos(t)}{(t - \pi)^2}$

(27.) $\lim\limits_{x\to 2} \dfrac{x^2 - 4}{\sin x}$

(28.) $\lim\limits_{t\to\infty} \dfrac{\arctan t}{t + 2}$

(29.) $\lim\limits_{x\to\infty} \dfrac{\ln x}{xe^{1/x}}$

In problems (30.) — (33.), use limits to determine which function dominates as $x \to \infty$.

(30.) $x^{0.2}$ and $\ln x$

(31.) $\ln(x^3 + 1)$ and x

(32.) x^6 and $e^{0.0001x}$

(33.) Two people are standing on the top of two equally tall buildings, separated by a gap of 200 feet. Person A drops an object which falls toward the street between the two buildings. Person B sees the object that falls as it forms an angle θ radians with a horizontal line, with vertex at the eye of B. Thus initially $\theta = 0$. If y is the distance traveled by the object in feet, obtain a formula for the rate of change of y with respect to the angle θ. You may assume that the object falls in a straight line.

(34.) An object near the surface of the earth, falls from rest so that it encounters air resistance directly proportional to its speed. In more advanced calculus courses it is shown that its altitude y feet above sea level at time t seconds is given by a formula of the form

$$y = -\frac{g}{K^2}\left(Kt + e^{-Kt}\right) + C$$

Here g, C and K are constants, with $K > 0$ and $g > 0$. The constants depend on initial position, proportionality constant of air resistance, and mass of the object.

(a.) Compute the velocity at time t.

(b.) (optional) Will the velocity ever reach $-g/K$? Why or why not?.