Math 215 Homework 1

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Note: All problems taken from the fifth edition of the textbook. If you have the fourth edition, you may do the same numbered problems as these which have different numbers, but are otherwise the same.

1.1.2 Solve using elementary row operations

$$\begin{aligned} 2x_1 + 4x_2 &= -4 \\ 5x_1 + 7x_2 &= 11 \end{aligned}$$

1.1.12 Solve using row operations on the augmented matrix

$$\begin{aligned} x_1 - 3x_2 + 4x_3 &= -4 \\ 3x_1 - 7x_2 + 7x_3 &= -8 \\ -4x_1 + 6x_2 - x_3 &= 7 \end{aligned}$$

1.1.16 Determine if the following system is consistent *without solving it.*

$$\begin{aligned} x_1 \qquad\qquad - 2x_4 &= -3 \\ 2x_2 + 2x_3 \qquad &= 0 \\ x_3 + 3x_4 &= 1 \\ -2x_1 + 3x_2 + 2x_3 + x_4 &= 5 \end{aligned}$$

Please turn over!

1.2.4 Row reduce the following matrix to reduced echelon form.

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$$

1.2.12,13 Find the general solutions of the systems whose augmented matrices are as below.

$$\left[\begin{array}{cccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{array}\right] \qquad \left[\begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

1.2.31 A system of linear equations with more equations than unknowns is sometimes called an *overdetermined system*. Can such a system be consistent? Illustrate your answer with a specific system of three equations in two unknowns.

1.3.1,2 Compute $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-2\mathbf{v}$ for the following examples:

$$\mathbf{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -3 \\ -1 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

Please turn over!

1.3.10 Write a vector equation which is equivalent to the following system of equations.

$$\begin{aligned} 4x_1 + x_2 + 3x_3 &= 9 \\ x_1 - 7x_2 - 2x_3 &= 2 \\ 8x_1 + 6x_2 - 5x_3 &= 15 \end{aligned}$$

1.3.18 (Similar to 1.3.16 in fourth edition) Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}.$$

For what values of h is $\mathbf{y}$ in the plane spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$?

1.3.25 Let

$$A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}.$$

Denote the columns of A by $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, and let $W = \text{Span}[\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3]$.

a. Is $\mathbf{b}$ in $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$? How many vectors are in $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$?

b. Is $\mathbf{b}$ in W?

c. Show that $\mathbf{a}_1$ is in W. *Hint:* Row operations are unnecessary here!

Please turn over!

1.4.12 Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

Write the augmented matrix for the linear system which corresponds to the matrix equation $A\mathbf{x} = \mathbf{b}$. Then solve the system and write the solution as a vector.

1.4.13 Let

$$\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$$

Is $\mathbf{u}$ in the plane in $\mathbb{R}^3$ spanned by the columns of A? Justify your answer!

1.4.17 Let

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$$

Does A have a pivot position in every row? *Hint:* Recall that a pivot position in a matrix A is a location in A that corresponds to a 1 (in the same position) in the reduced echelon form of A.

Please turn over!

1.4.18 Let

$$B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$$

Do the columns of B span $\mathbb{R}^4$? *Hint:* For this you will need a theorem from the notes which you should cite. The previous question may also be of help!

1.5.3 Determine if the following system has a solution. You should try to use as few row operations as possible.

$$\begin{aligned} -3x_1 + 5x_2 - 7x_3 &= 0 \\ -6x_1 + 7x_2 + \ \ x_3 &= 0 \end{aligned}$$

1.5.5 Write the solution of the following homogeneous system in parametric form.

$$\begin{aligned} x_1 + 3x_2 + \ \ x_3 &= 0 \\ -4x_1 - 9x_2 + 2x_3 &= 0 \\ -3x_2 - 6x_3 &= 0 \end{aligned}$$

Please turn over!

1.5.7 Let

$$A = \begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix}$$

and describe all solutions of the matrix equation $A\mathbf{x} = \mathbf{0}$ in parametric form.

1.5.13 Suppose the solution set of a certain system of linear equations can be described as $x_1 = 5 + 4x_3$, $x_2 = -2 - 7x_3$, with x_3 free. Use vectors to describe this set as a line in $\mathbb{R}^3$

1.5.17 (1.5.15 in 4th Ed.) Describe and compare the solution sets of
$x_1 + 9x_2 - 4x_3 = 0$ and $x_1 + 9x_2 - 4x_3 = -2$