MTH 436/Final Exam

Take home part-Due 5/14/2004
Spring 2004
University of Rhode Island
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1. Suppose that $u_k \geq 0$ and that the series $\sum_{k=1}^{\infty} u_k$ diverges.
(a) Prove that the series $\sum_{k=1}^{\infty} \dfrac{u_k}{1+u_k}$ also diverges.
(a) Prove that the series $\sum_{k=1}^{\infty} (e^{u_k} - 1)$ also diverges.

2. Let O be an open subset of R^n and suppose that the function $f : O \to R$ is continuous.
(a) If a and b are numbers with $a < b$, prove that the set

$$\{\overrightarrow{u} \in O | \ a \ < f(\overrightarrow{u}) < b\}$$

is open in R^n.
(b) If a and b are numbers with $a < b$, prove that the set

$$\{\overrightarrow{u} \in R^n | \ a \ \leq f(\overrightarrow{u}) \leq b\}$$

is closed in R^n.

3. Use the second derivative test to find all points of minimum and maximum of the function

$$f(x, y) = ax^2y^2 + b(x^2y + xy^2) + c(x^2 + y^2) + dxy.$$

4. Consider the system of equations:

$$\begin{aligned} x^5y + y^7z + z^{11}u + u^{14}v - 2v^{16}x = 2, \\ y^5x + x^{11}z + z^{10}u - 5u^7y + v^{12}z = -1, \end{aligned}$$

at the point $(1, 1, 1, 1, 1)$.
(a) Find all pairs of variables that can be expressed as the function of remaining three variables in a neighborhood of $(1, 1, 1)$.
(b) Evaluate partial derivatives at the point $(1, 1, 1)$.