MTH 132

Quiz V

Name:

Show all your work.!

(1) The equations below give the rates of growth of two populations, x and y, measured in thousands.
   (a) Describe in words what happens to the population of each species in absence of the other.
   (b) Describe in words how the species interact with one another. Give reasons why the populations might behave as described by the equations. Suggest species that may interact the same way.

(i)

$$\frac{dx}{dt} = 0.01x - 0.05xy$$
$$\frac{dy}{dt} = -0.2y + 0.08xy$$

(ii)

$$\frac{dx}{dt} = 0.01x - 0.05xy$$
$$\frac{dy}{dt} = 0.2y - 0.08y$$

a) (i) The population of x increases in absence of y
The population of y decreases in absence of x.

(ii) The population of x increases in absence of y
The population of y increases in absence of x.

(b) (i) Species x will decline when y is introduced
Species y will do better when x is introduced
Predator / pray relationship

(ii) Species are in competition with one another

(2) Compare the diseases modeled by each of the following differential equations with the flu model that we studied in class. Match the set of differential equations with one of the following statements. Write a system of differential equations corresponding to each of the unmatched statements.

(a)

$$\frac{dS}{dt} = -0.04SI$$
$$\frac{dI}{dt} = -0.04SI - 0.2I$$

(b)

$$\frac{dS}{dt} = -0.002SI$$
$$\frac{dI}{dt} = 0.002SI - 0.3I$$

(c)

$$\frac{dS}{dt} = -0.03SI$$
$$\frac{dI}{dt} = 0.03SI$$

(i) More infectious; infecteds removed more slowly.
(ii) More infectious; infecteds removed more quickly.
(iii) Less infectious; infecteds removed more slowly.
(iv) Less infectious; infecteds removed more quickly.
(v) Infecteds never removed.

a → i
b → iv
c → v

ii) $\frac{dS}{dt} = -.05SI$

$\frac{dI}{dt} = 0.05SI - 0.4I$

iii) $\frac{dS}{dt} = -.001SI$

$\frac{dI}{dt} = .001SI - 0.1I$

(3) Find the sum, if it exists

(a)

$$3+\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\cdots+\frac{3}{2^{10}}$$

(b)

$$1000+1500+2250+3375+5062.5+\cdots$$

(c)

$$200+100+50+25+12.5+\cdots$$

(d)

$$500+500(1.6)+500(1.6)^2+\cdots+500(1.6)^{15}$$

a) $3\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^{10}}\right) = 3\left(\frac{1-(1/2)^{11}}{1-1/2}\right)$

$a=3$

$r=1/2$

$= 6\left(1-(.5)^{11}\right)$

b) $1000\left(1+1.5+(1.5)^2+(1.5)^3+(1.5)^4+\cdots\right)$

The series diverges since $r=1.5$

c) $200\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{2^4}+\cdots\right)$

$200\left(\frac{1}{1-1/2}\right) = 400$

d) $500\left(1+(1.6)+(1.6)^2+\cdots+(1.6)^{15}\right)$

$= 500\left(\frac{1-(1.6)^{16}}{1-1.6}\right)$