MTH 243

Quiz IV (Take home)

Name:

Show all your work.!

(1) Find the equation of the tangent plane at the given point

$$z = e^y + x + x^2 \quad \text{at the point} \quad (1, 0, 9)$$

(2) An unevenly heated plate has temperature $T(x, y)$ in centigrades at the point (x, y). If $T(2, 1) = 135$ and $T_x(2, 1) = 16$, $T_y(2, 1) = -15$ estimate the temperature at the point $(2.04, 0.97)$

(3) Find the gradient at the given point

$$f(x, y) = \sqrt{\tan x + y} \qquad \text{at } (0, 1)$$

(4) Find the directional derivative $f_{\vec{u}}(1, 2)$ for the function $f(x, y) = \sin(2x - y)$ with $\vec{u} = (3\vec{i} - 4\vec{j})/5$

(5) Let $f(x, y) = x^2 + \ln y$
   (a) Find the average rate of change as you go from $(3, 1)$ to $(1, 2)$
   (b) Find the instantaneous rate of change of f as you leave the point $(3, 1)$ heading toward $(1, 2)$.

(6) Find the directional derivative using $f(x, y, z) = xy + z^2$. At the point $(2, 3, 4)$ in the direction of a vector making an angle of $3\pi/4$ with $\nabla f(2, 3, 4)$.

(7) The surface S is represented by the equation $F = 0$ where $F(x, y, z) = x^2 - (y/z^2)$.

(a) Find the unit vectors $\vec{u_1}$ and $\vec{u_2}$ pointing in the direction of maximum increase of F at the points $(0, 0, 1)$ and $(1, 1, 1)$ respectively.

(b) Find the tangent plane to S at the points $(0, 0, 1)$ and $(1, 1, 1)$

(c) Find all points on S where a normal vector is parallel to the xy-plane.

(8) Find $\partial z/\partial u$ and $\partial z/\partial v$. The variables are restricted to domains where the functions are defined.

(a) $z = xe^y, \qquad x = \ln u, \quad y = v$

(b) $z = xe^{-y} + ye^{-x}, \qquad x = u\sin(v), \quad y = v\cos(u)$

(9) Find the critical points and classify them as local maxima, local minima, saddle points or none of these

(a) $f(x, y) = x^3 - 3x + y^3 - 3y$

(b) $f(x, y) = e^{2x^2+y^2}$

(10) Use Lagrange multipliers to find the maximum and minimum values of f subject to the given constraint if such values exist.

$$f(x, y) = x^3 + y, \qquad x + y \geq 1$$